NUMERALS AND NUMBERS, INDEX NUMBERS AND COEFFICIENTS IN INTER-, MULTI-, TRANS-, AND CROSS-DISCIPLINARY RESEARCH'S LANGUAGE

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Abstract. *Numerals (digits), numbers, index numbers and coefficients are sometimes considered some usual mathematical expressions from a fundamental scientific language but also elements of the modern inter-, multi-, trans-, and cross-disciplinary research. In their first signification, numerals (digits), numbers, index numbers (IN) and coefficients are the result of a long process of evolution and a vast typology is a key to understanding all of their senses. In their second signification numerals (digits), numbers, index numbers and coefficients are the components of modern research and education based on an inter-, multi-, trans-, or cross-disciplinary approach. After a brief introduction underlying three vital questions for any researcher, this paper details the numerous family of numerals (digits), numbers, index numbers and coefficients, with some similarities and differences in their usual sense in mathematics and in any inter-, multi-, trans-, and cross-disciplinary research. Some final remarks are important for a better understanding of mathematical language in modern research.*

Keywords: *quantity, measure, numeral (digit), number, index number (IN), coefficient, elasticity, correlation, associations, inter-, multi-, trans-, and cross-disciplinarity.*

1. INTRODUCTION

During a quantitative scientific investigation of socio-economic or demographic real life, in a usual or even unusual inquiry, where organizing observed data into meaningful structures becomes essential, researchers must answer the minimum next three major questions about numerals (digits), numbers, index numbers and coefficients: I) Can one research be finalized as a real investigation without defining, redefining and even underlining the most important differences between numerals (digits) and numbers or between index numbers and coefficients, and especially without a methodology of practical validation of these primary instruments etc.? II) Can the same research offer a qualitative conclusion or a final remark without the help of measured variables, or without some analyses based on associations or correlations between the same phenomenon's variables? III) Can researchers obtain some reasonable or valid results and especially a short-term foresight without some specific methods or models, based on using sometimes even only a few numerals (digits), numbers, index numbers and coefficients? These questions and their specific answers perturb the way of thinking of any researcher and he or she must find an answer before starting his or her new research.

One can now imagine these expanded aspects in inter-, multi-, trans-, and cross-disciplinary research when all these are multiplied and hence significantly different in importance and impact. In this way, the introductory approach of this article can be considered already a complete one. In the central section of the paper, the necessary notions are first all defined and detailed to specific inter-, multi-, trans-, and cross-disciplinary research. Finally, all the researchers must be connected to the whole investigation in a characteristic team system, and the results many times expressed in numerals and numbers, index numbers and coefficients. Some final remarks or conclusions are drawn in the end, emphasizing the main gains of quantifying results and validating or not initial hypotheses, thus developing a wider or better understanding of reality.

2. NUMERALS (DIGITS) AND NUMBERS' EVOLUTIONS AS ESSENTIAL TERMS IN MATHEMATICAL LANGUAGE

What really are numerals (digits), numbers, index numbers and coefficients in Mathematics results or in its scientific language? Semiotics, born from "*semesion*" or sign, in the Greek language offers a credible answer to this question [1]. This answer is only the word "*signs*". Semiotics, as modern science, developed from the study of meaning in language, can be applied to entire text or only to a single word. In the same old Greek language "*semainon*" was used as a signifier and "*semainomenon*" meant signified or indication. In modern Semiotics are reunited Semantics, Syntactics (Syntax), and Pragmatics (fig. no.1):

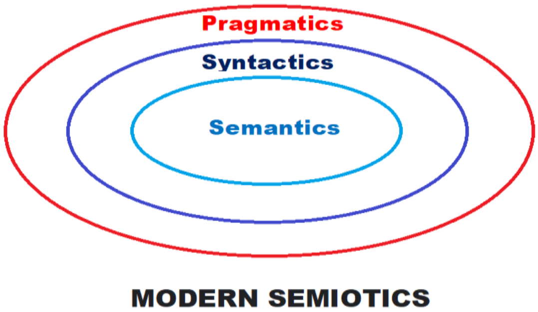

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Fig. 1. Structure of Modern Semiotics - base layers

Semantics means the associations of signs to what they stand for, meanwhile Syntactis offers formal or structural relations between the same signs, and Pragmatics underlines the link of signs to interpreters. The initiators of this difficult and more and more extended science, named Semiotics remain Ferdinand de Saussure [2], and Charles Sanders Peirce [3]. The first, a well-known linguist from Switzerland, describes the new science as a theory of sign and "baptize" it as Semiology. Saussure's Semiology is based on the principle that "*emphasized language as a system of sign, and besides language, there are many other sign systems that exist in the world of mankind*." [2]. The second scientist was an American philosopher well-known as a pioneer of pragmatism doctrine, who named Semiotics the new science"*synonymous with the concept of logic that focuses on the knowledge of human thinking process* [3]," and thus main principles containing Peirce's theory, were *"the human mind and sign boundaries, the three-dimensional system (triadic/trichotomy) and the relativity regarding the three typologies or taxonomies of signs: i) icon; ii) index,* [including Index Number–IN and Coefficient-C]*; iii) symbol* [3]."

But science's evolution is more complicated and sometimes a simplifying personality must solve or complicate everything. This was the case with Umberto Eco for Semiotics. In his *Theory of Semiotics*, Umberto Eco is connected with all aspects that can be taken as a sign, "*generalized as everything which can be taken as significantly substituting for something else*" [4]. Sometimes that "*something else*" does not exist exactly at the same time when the sign represents or replaces its position, and therefore, Umberto Eco often refers to the *Theory of Semiotics* as a theory of lie (deception) because it can be used for misleading or deceiving the others [4].

Can a researcher say about numerals (digits), numbers, index numbers and coefficients are signs used for misleading or deceiving some other researchers? Well, in Mathematics' results or in its scientific language when this aspect appears soon or later must be present a classic *erratum* or a calculus must be declared affected by real error …

But all definitions can clarify better any of the concepts, including numerals (digits), numbers, index numbers and coefficients in mathematics. Numerals are just simple "*symbols or collections of symbols used to represent small numbers, together with systems of rules for representing larger numbers*." [5] Are numerals real numbers or are they not? The earliest numerals were either simple notches on a stick or rudimentary scratches on a stone, marks on a pottery piece etc. There was no real necessity for written numerals until the beginning of civilized times. Initial, the first numeral designates a number of objects being necessary as a symbol for the small numbers, and thus for abstract notions like one or two humans someone needs independent signs, and particular representations and all of these probably appeared very late [6].

Numbers are complex constructions if one tries to compare numbers with numerals. Numerals are the original bricks for all these constructions named numbers, together with some rules for obtaining a correct solution on different landscapes or streets, and without restriction to the decimal system. After these rules redefine words as digit and numeral in any base (binary, senary, octal, decimal, hexadecimal, etc) and any systems of numeration (the Hindu-Arabic system, the Babylonian system, the Greek system, the Chinese system), anyone can build numbers from numerals (digits).

The first numbers were usual arithmetic values used for representing the quantity**,** but all evolved in measuring, in making calculations, in counting, in keeping things in order, in indexing, etc. Initial or innovative numbers need numerals (digits or symbols in a logical manner) or a specific number system. The first numbers were natural, extracted from nature. Beyond the natural numbers, there are other types of numbers that have emerged throughout history in order to express situations and solve problems for which the first were insufficient [7].

From a semantical point of view, between numerals (digits) and numbers, there is not only one connection but also there are different meanings. Thus, numbers sense can be similar to "semainon", being used as a signifier, and numerals (digits) meaning sometimes could be signified or indication ("semainomenon"). A long period, in the old Greek mathematic language and another even longer period after them, "numbers" were just integers, constructed from "numerals" (digits), from 1 to 9, not including 0 ("zero", "sunya", "cipher", "sifr", "zephyrum" etc.). Zero was taken from the Babylonian origin in the Indian mathematic language and somehow purified through Chinese and Egyptian logic and pragmatic minds. "Zero" became a very important or special numeral, defining not only the absence of all sizes or quantities but also a unique landmark or a specific border placed between all negative numbers and all positive numbers. Finally, zero is a symbol used to represent the absence or the emptiness.

Without a doubt, all of the researchers can remember integers, rational numbers, real numbers, in fact following numbers evolutions to integer, or to remain rational and why not to maintain reality for a brief or a long period of time. In the case of real numbers, it means probably a too long period of time: "*Euclid and Archimedes had together a good intuitive grasp of the concept of real number, and they had a theory of proportions when discussing such things as lengths and areas, and* *what we call irrational numbers. This theory of proportions was used up to the 18th century"* [1].

But what means time in mathematics language evolution and especially in the theory of numbers and as a proof, how many of us really know that there are newborn numbers [7] baptized as "*happy numbers*" or "*palindromic numbers*"?

The major difference between a numeral (digit) and a number remains that a numeral (digit) is a single numerical symbol, whereas a number can contain a single numeral (digit) or a combination of more numerals (digits). Anyone can try immediately another question like: What's the difference between a numeral and a digit? It might look easy to answer it, but in truth, it's indeed a very difficult way to do it, without mathematical rules. Digit in the dictionary is always defined as approximately unclear like: "*any one of the ten numbers 0 to 9"* (*Cambridge Academic Dictionary: Cambridge University Press,* and *the same digit disappears and* becomes a little detailed as an Arabic numeral in Merriam-Webster: "*any of the number symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9*" [8]. Some mathematical dictionaries use other definitions as: "*A digit is a symbol (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) used to write the numbers.*" (*Icoachmath.com Math_dictionary*) [9] The basic definition of digits became somehow a similarity with numerals, sometimes a profound one, between these two mathematical terms: *"any of the numerals from 0 to 9, especially when forming part of a number"* or "*elements used in forming numbers in other number systems*" [10]. It means one can use a group of numerals for written digits in a number, based on a mathematical system, named *decimal system of numerals* (nine digits and a zero). The first synonym for digit still remains **numeral**, followed in a strange manner by *number*, figure, integer etc. That is because the digit's notion implies duality, "*modern digit*" having the meaning when of a number and when of a column defined through ranges of equal value (i.e. "7-digit" incomes as a simplified expression for very high salaries and thus describing how many columns someone really needs to write the number and not a real wage).

"Modern digit" is still a numeral or rather a symbol and less a number, having the quality of a substitute, emphasizing a numeric value level and changes, anytime the same symbol is put in a different place when someone writes numbers.

From the beginning, a number transformed a measurement or a quantitative idea from human minds into reality. During the last three or four millennia, the entire humanity used numbers to think, discuss or write about almost everything, the numbers being usual, unusual or even unique like π and ε, the special number being placed always behind mathematical ideas. The most important association between number and numeral (digit) remains that of their mandatory coexistence in mathematical language. Thus, a mathematical idea, called a number, can become a written reality only with the help of a numeral (digit). Dominant relativity, as the law of numerals' or digits' association, generated a type of continuum of symbols, between idea and reality, but also reciprocally. Similar to letters making words in literature, numerals (digits) stand for any idea of a number as an excerpt initially from reality, and finally from a mathematical calculus (algorithm etc.). From the Latin language, the digit was translated with a similar sense of a finger in mathematical language. Romans were also those who tried really hard to unify symbols from the Latin language and used letters as substitutes for numerals (digits). But as with many other unsuccessful attempts, a generalized compromise was again entirely refused by mathematical language throughout its history. Numerals (digits) and numbers remain even now the "*foundation of mathematics*" and the theory of numbers became the "*queen*" of mathematical theories [7]. A synthesis of the most important aspects characterizing numerals (digits) and finally numbers is detailed below (fig. no. 2):

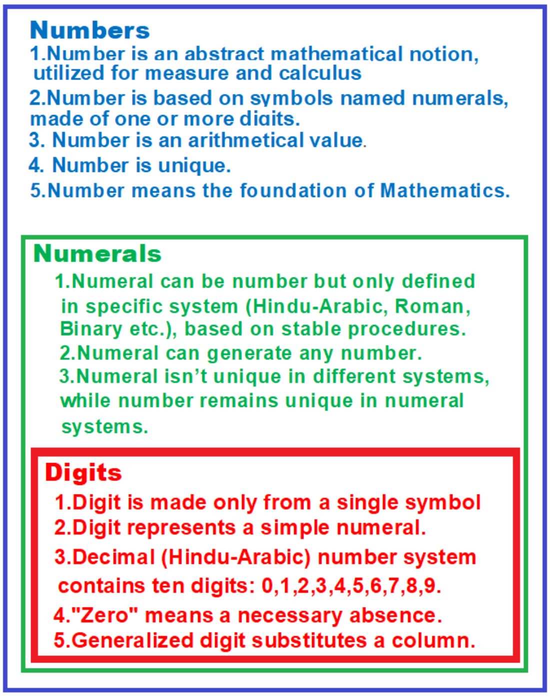

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Fig. 2. Structure of mathematical numbers

The mathematical language was continuously enriched and diversified by the typology of numbers, which has developed a family into more and more complex numbers (even hypercomplex numbers). Typology has become more important for numbers than arithmetic operations.

In fig. no. 3 some of the new classes of numbers are several classes of numbers that are detailed with a purely demonstrative role as a proof of a continuous introspection in the complex universe of the theory of numbers, extended from numeral (digit) to number, regrouped criterionically in systems and under the influence of inter-, multi-, trans-, and cross-disciplinary researches:

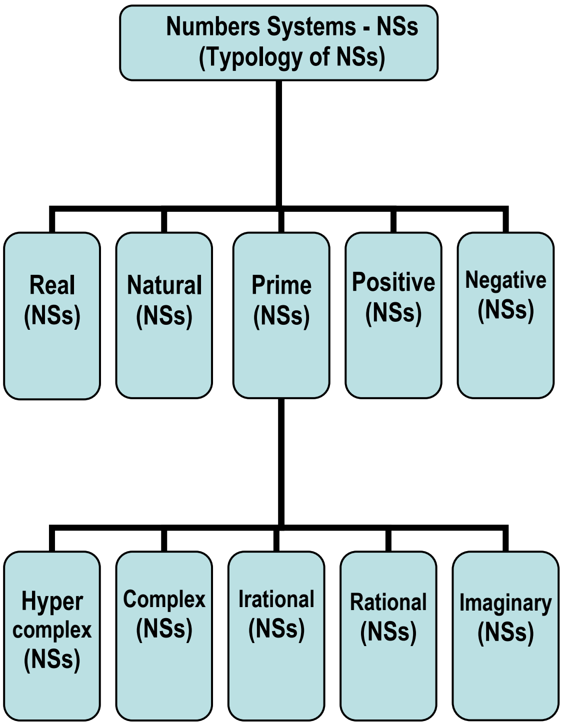

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Fig. 3. Typology of Number Siystems (NSs)

In the modern language of mathematics, any real number from any NSs can be described as an (in)finite set of numerals (digits) or as a numeral's sequence, where the uniform distribution becomes closer to reality, and the natural density tends to be approximately equal anywhere from algebraic statistics to financial econometrics [11, 12].

3.INDEX NUMBERS(INs) AND COEFFICIENTS (Cs) AS RESULTS FROM INTER-, MULTI-, TRANS-, AND CROSS-DISCIPLINARITY

Index Numbers (INs) and Coefficients (Cs) are coming not only from Peirce's theory of the *"three typologies of signs: i) icon; ii) index* [INs & Cs]; *iii) symbol* [3]," but also from inter-, multi-, trans-, and cross-disciplinary researches [13]. The modern numbers' diversification is based more on the experience from these four research's attempts or forms of crossing the borders of any isolated discipline or any unidisciplinary attitude: inter-, multi-, trans-, and cross-disciplinarity, broadening the perspective of knowledge.

In any multi-disciplinary research "*the subject under study is approached from different angles, using different disciplinary perspectives*," multiplying results and implicitly numbers, but even so "*neither the theoretical perspectives nor the findings of the various disciplines are integrated into the end*" [14], meanwhile any interdisciplinary research "*creates its own theoretical, conceptual and methodological identity, and as consequence even its results* [including numbers] *in an interdisciplinary study being more coherent and integrated*" [14]. Any multidisciplinary research remains within the boundaries of initially connected disciplines, based on their specific knowledge and way of thinking. while any interdisciplinary investigation "*analyzes, synthesizes and harmonizes links between disciplines into a coordinated and coherent whole*" [15].

Cross-disciplinarity can generate also new numbers or classes of numbers by applying methods from a well-known science in various other newer or not sciences or confers values initially unsuspected to some numbers by changing the domain, through the specific measurement of new phenomena, based on the units of measure or even because of some pragmatic details.

Starting from its own three methodological axioms, and in conformity with Basarab Nicolescu's books and papers, trans-disciplinarity multiply essential numbers and enrich also their typology, having access to: "*different levels of Reality of the Object and of the Subject (ontological axiom), also to the passage from one level of Reality to another, ensured by the logic of the included middle (logical axiom), and finally to the structure of the totality of levels of Reality, as complex structure, where every level is what it is because all the levels exist at the same time (complexity axiom)*" [16, 17, 18, 20].

Index Numbers (INs) and Coefficients (Cs) are the real expressions of modern multiplying sense or significations of numbers in the mathematical language under the influence of inter-, multi-, trans- and cross-disciplinarity. Index numbers (INs) are the measures of average (overall) changes in a group of related variables, having a constructive role in homogenization in many heterogeneous statistical populations. You cannot value an average of values placed in extremes, such as the values of the prices of different products on a global market, but when you use indices, this population of the resulted indices suddenly becomes homogeneous and close to normal and continuous distributions

INs appeared more than three centuries ago, "*from the remarkable curiosity of an English cleric and political arithmetician, William Fleetwood, to measure the real evolution of the purchasing power of the population in England, which is research published in Chronicum preciosum*" [20]. INs have gradually occupied the new interdisciplinary and multidisciplinary researches in economics, health, sociology, demography, management etc. In Statistics, INs forms a specific method of analysis and covered more and more domains, based on cross-disciplinarity. Transdisciplinary approaches based on INs are useful as solutions for transforming any heterogeneous population or database into a homogeneous one because of their capacity of measuring flexibility in time, space or structure, not only more properly, but also more precise. INs are indeed modern numbers because can guarantee that certain conditions will be better fulfilled in general research like the ideas of stability, order, balance, substitution, hierarchy, aggregation, disaggregation, and thus generating the concept of INs' system, These new statistical numbers named INs can measure now almost everything and everywhere.

Coefficient as a mathematical notion was introduced by French mathematician François Viète (1540-1603), influenced by Modern Latin, after a brief period when it has been initially written as "*co-efficient*" and represented "*a number or a letter which unites in action with something else to produce a given effect,"* The first sense of the concept was "*acting in the union to the same end*" [21]

The coefficient is a numerical or constant quantity placed before and multiplied by the variable in an algebraic expression, and a modern number helps the researcher to valorize a measure of some property or characteristic (as of a substance, device, or process). Any coefficient underlines if the value of an independent variable tends to increase/decrease, being useful in many investigations, referring to economic elasticity, industrial flexibility and rigidity, metal expansion, function value, etc.

Any coefficient as any number is unique but only up to a scaling operation, so the magnitude of some newborn coefficients can remain arbitrary or exposed to errors or omissions (i.e. final limits in elasticity of demand in fig. no. 4)

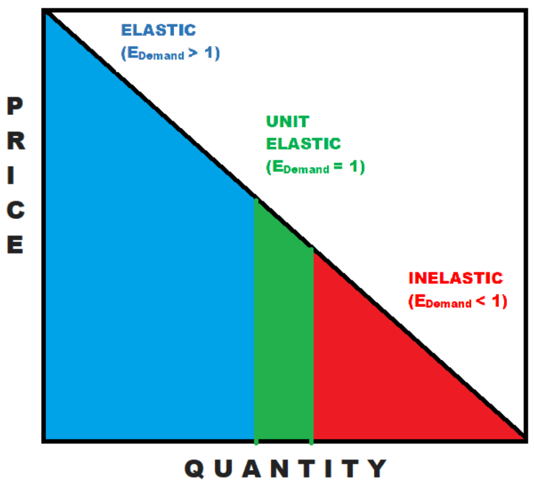

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Fig. 4. An example of Cs' evolution and limits

The difference between the two initially exclusively spatial, temporal and structural statistical constructions called classical IN (i.e. Laspeyres or Paasche type) can be found during a mathematical demonstration in the very essence of the elasticity coefficient. Thus, statistical thinking passes mathematically from the INs method to the Coefficient (Cs) method of elasticity [22].

The relationship between the index and the coefficient identifies the essence of the phenomena represented, namely the economic elasticity (of demand or supply) and represents a statistical correlation between two pure INs' and Cs' methods, using the new numbers as original interrogative and demonstrative cycles of cross-disciplinary thinking [22, 23]. If the index (under the name Index-Number) was used to measure a change of a quantitative nature, which could not be directly observed (Bowley, 1920) or to show either the increase or the decrease of a fairly large value difficult to measure accurately (Edgeworth, 1925), the INs Method was and is an insufficiently exploited method directly and the encounter with the elasticity coefficient method was not accidental (Bortkiewicz, 1932), but represented a creative association that generated both new numbers as well as original inter-, multi- and transdisciplinary solutions for their generation [24,25].

4. SOME FINAL REMARKS

Day after day, the language of mathematics develops more and more from numeral (digit) to numbers, and more recently, through "*happy numbers*" or "*palindromic numbers*", it gets closer and closer to the richness of any language. To have a more comprehensive view of the future of numbers in mathematical science, and their cohesion or unity as a universe structured by systems, sets, classes, groups etc. and multiplied through inter-, multi-, trans-, and cross-, disciplinarity we need many more theoretical researches and practical investigations. However, even this paper can be a good example, showing that overcoming the fragmentation created through scientific questions remains a valuable solution that requires deep foundations and overall evolution analysis, especially in the form of more clearly defined mathematical terms in mathematical language. "Quantitative description of phenomena, perceived by Auguste Comte and many others after him as evidence for exactness, can impress only novices" [13], but I believe that without exactness or precision and stable results or numbers there is no mathematical science or mathematical language.

Of course, the objective key will remain forever in the balance between mathematical ontology and mathematical epistemology, and also in the correspondence between language structures that define and describe entities and parts of the whole (logical, and mathematical). I think that more papers and discussions about mathematical language and major mathematical concepts must be experienced, promoted, encouraged, published, integrated into research teams, and exchanged between the same research teams offering more flexibility, selectivity, exploratory analysis, innovative value, even creativity, more adequate scientific vision, relevancy, etc.

As the point elasticity or the arc elasticity can vary in value depending on the starting point, always numbers can evolve in different systems, sets, and classes of high or low volumes, so the numbers will forever remain unique for any mathematician.

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